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The research findings during this grant period fall into two categories. The first category includes methods for repairable systems where repeated failures are modeled by either a nonhomogeneous Poisson process or else a compound nonhomogeneous Poisson process. The second category includes methods for nonrepairable systems. Papers in this second category are often useful in the analysis of repairable systems when applied to first-failure data. Such an analysis is often useful in the preliminary analysis of repairable systems when only early failure data is available.

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"RELIABILITY ASSESSMENT FOR ONE-SHOT DEVICES BASED ON REPEATED SAMPLES"

FINAL REPORT OF RESEARCH ACCOMPLISHMENTS
FOR GRANT NUMBER AFOSR 84-0164

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SUMMARY - The research findings during this grant period fall into two categories. The first category includes methods for repairable systems where repeated failures are modeled by either a nonhomogeneous Poisson process or else a compound nonhomogeneous Poisson process. The second category includes methods for nonrepairable systems. Papers in this second category are often useful in the analysis of repairable systems when applied to first-failure data. Such an analysis is often useful in the preliminary analysis of repairable systems when only early failure data is available.

Denote by $X(t)$ the number of failures of a repairable system in time t. It is well known that under certain conditions $X(t)$ is a Poisson process with mean function $m(t) = E[X(t)]$. If $m(t)$ is continuous, the process is called a regular Poisson process. The derivative of $m(t)$, $\nu(t) = \frac{d}{dt}m(t)$ is called the intensity function. Perhaps the best known case is a homogeneous Poisson process (HPP) in which case the intensity function is constant. Often the reliability criteria for repairable systems is based on this assumption, such as in MIL-STD-781C. However, much of the recent interest has involved processes with nonconstant intensity functions or nonhomogeneous Poisson processes (NHPP's). An NHPP is capable of modeling changes such as deterioration in an aging system, whereas an HPP can not account for such behavior. Two common examples are the Weibull process and the log-linear process.

A compound (mixed) Poisson process offers an alternative to the NHPP. Such a compound process occurs when the population consists of Poisson distributed individuals, but the intensities vary among individuals.

The following list gives the titles and main results of sixteen papers which were produced during this grant period:

1. "On the Asymptotic Behavior of the Mean Time Between Failures for Repairable Systems". The mean time between failures (MTBF) is constant in the HPP case, which provides a convenient interpretation of reliability criteria, however it is not clear what concept should take the place of the MTBF in the case of an NHPP. A number of generalizations of MTBF can be found in the literature, although

many of these are equivalent in the case of a regular NHPP. In this paper, two possibilities are studied. The simplest is the reciprocal of the intensity function, $1/\nu(t)$. The other is the mean forward recurrence time, $E[W_t]$, where $W_t = T_{X(t)+1} - t$, which is equivalent to the conditional mean of the time between failures given the time of the last failure when the process is regular. It was shown that these are asymptotically equivalent for large t, for several of the more common NHPP's. This work will be continued with the objective of finding more general, but realistic, conditions under which these concepts are asymptotically equivalent.

2. "Tests for an Increasing Trend in the Intensity of a Poisson Process : A Power Study". This paper was concerned with a comparison of several tests for testing the hypothesis of an HPP versus the alternative of an NHPP with an increasing intensity function. The study included the well known Laplace test, the most powerful unbiased test for the shape parameter of a Weibull process, the likelihood ratio test against arbitrary NHPP's with increasing intensity, two common nonparametric tests for trend, and a test based on an F-statistic. A variety of alternatives were considered, including log-linear rate, Weibull rate, and logarithmic rate. Based on simulation, the test for the shape parameter of a Weibull process was found to perform quite well for the alternatives considered.

3. "Goodness-of-fit Tests for the Weibull Distribution with Unknown Parameters and Heavy Censoring". In this paper, goodness-of-fit tests were considered for testing the two-parameter Weibull distribution based on type II censored sampling with both parameters unknown. Some extremely heavy censoring levels were considered which are useful when analyzing in-service field data with a large population and a small number of early failures. Several well-known nonparametric tests were considered along with the two-sided Mann-Scheuer-Fertig test.

4. "Statistical Analysis of a Compound Exponential Failure Model". In this paper, statistical analyses for the parameters of a mixed or compound exponential failure model were discussed. Tests of hypotheses were developed and the construction of confidence limits were discussed. This type of model would be of interest when components which have exponential failure times are operated under diverse environmental conditions. In this situation a component selected at random could be regarded as having a random hazard rate, and consequently the component would have a compound exponential failure distribution. One variation on this would be to consider compounding with a repairable system. In particular, a compound Weibull process would be useful. Some preliminary results have been obtained for such a model and further work will be carried out.

5. "Tests of Equal Gamma Distribution Means with Unknown and Unequal

Shape Parameters". An approximated F-test was provided for testing the equality of means for two gamma distributions when the shape parameters may be unknown and unequal. The approximation was studied by simulation and asymptotic results were also derived. An approximate F-test for equal shape parameters was also studied. Although the gamma distribution is an important model in reliability and life-testing, statistical methods are generally scarce, particularly in the case of two-sample problems.

6. "Approximate Tolerance Limits and Confidence Limits on Reliability for the Gamma Distribution". This paper considers methods for constructing tolerance limits, which is particularly troublesome when sampling from a gamma model since it cannot be transformed to a location-scale model. Two approximate methods are described and studied by simulation methods. The construction of confidence limits for reliability in this situation is also considered.

7. "Approximate Distributional Results Based on the Maximum Likelihood Estimators for the Weibull Distribution". This paper considers some convenient approximations for the distributions of pivotal quantities which are based on the maximum likelihood estimators of the parameters. These provide an improvement over some earlier approximations which appeared in literature, and they are less dependent on the availability of special tables.

8. "Prediction Intervals of the Gamma Distribution". This paper provides a method for constructing an approximate prediction interval for a single future observation or for the average of m future observations from a gamma distribution when both parameters are unknown.

9. "On the Mean Time Between Failures for Repairable Systems". In this paper, a general condition is given under which two important generalizations of the mean time between failures are asymptotically equivalent. It is suggested in this paper that inferences on the mean time between failures can be developed on the basis of the reciprocal of the intensity function for many types of nonhomogeneous Poisson processes, including the Weibull Poisson process. The only known method for such inferences applies when the truncation time is the same as the mission time.

10. "Statistical Analysis of a Compound Weibull Model for Repairable Systems". A compound Poisson distribution is sometimes used as an alternative to the Poisson distribution for count data. This paper considers a more flexible model where system failures are distributed according to Weibull Poisson processes with different intensity parameter values, which vary according to a gamma distribution. In this paper methods are developed based on data from several systems which are observed over time intervals of equal length.

11. "Reliability Test Plans for One-Shot Devices Based on Repeated Samples". This paper deals with the problem of devices which will either function properly or fail when tested. The term "one-shot" refers to a device which is destroyed when it functions properly. Such testing differs from the standard life-testing situation in that the data is dichotomous (go or no go) data rather than data obtained by measuring a continuous variable such as failure time or stress level. Suppose, at some point in time, it is desired to demonstrate that a certain type of device satisfies a prescribed reliability requirement at a specified level of confidence. Of course, if several devices are tested independently, then the number of successful tests will be a binomial random variable with parameters n, the number of tests, and p, the probability that a device will work properly (i.e. the "reliablity" of the device). If one or more sets of "past" data are available, an obvious question would be: Can such data be used in order to demonstrate the desired reliability requirement with a smaller sample size for the current test program? For example, suppose annual samples are taken from a stockpile of weapons in order to insure that the ability of the weapons to function has not decreased markedly with age. If the reliability can change from one test to the next, it would appear that some model assumptions would be necessary in order to utilize information from previous samples to enable future sample sizes to be reduced. The present approach is to consider the probability of successful operation of a device at time t, to be a function of t, p(t) and to make reasonable model assumptions in terms of this function. The paper focuses on two models. In particular,the Weibull and linear degradation models are considered. The emphasis in this paper is on test plans for periodic testing of highly reliable one-shot devices. Methods for determining the smallest sample size at each stage of sampling, based on earlier test data, in order to maintain specified reliability criteria, are derived.

12. "Inferences on N in a Binomial Distribution". A method is described for finding the distribution of an estimator for the parameter N of the binomial distribution in order to construct confidence bounds for N. Such results can be applied in reliability problems. For example, suppose that the population consists of a large number of devices, some of which are defective, but in which the number, N, of defective devices is unknown. Another example, involving software reliabilty, would relate to the development of a large program in which there are an unknown number of errors or "bugs". In recent years a number of papers have considered the problem of estimating the parameter N in a binomial distribution with unknown success probability p. Primarily, they consider the method of moments estimator and the maximum likelihood estimator of N based on a random sample of size k from a binomial distribution. They also

consider certain modifications of these estimators in an attempt to obtain more "stable" estimators. Many of the recent papers have been concerned with this problem of the relative instability of the estimators of N. Our purpose, in this paper, is to consider lower confidence bounds for N based on the conditional maximum likelihood estimators. Using a partition generating algorithm, tables of lower confidence bounds are provided for a range of values, illustrating the nature of the problems involved in estimating the parameters.

13. "A Study of Parametric and Distribution-Free Tolerance Limits for the Gamma Distribution". This paper deals with the robustness of a certain approximate tolerance limit for the gamma distribution under a Weibull alternative. The robustness of a Weibull tolerance limit under a gamma alternative is also studied. It is found that neither method is very robust, but that the Weibull limit is conservative for the gamma distribution with shape parameter greater than one. The amount of precision lost by using the robust distribution-free method based on an order statistic, compared to using the parametric gamma method is also studied.

14. "Tests for Positive Jumps in the Intensity of a Poisson Process: A Power Study". Tests for the hypothesis of a constant intensity against the alternative of an increasing intensity in a NHPP were considered. Attention was focused on step-function alternatives and tests were designed for such alternatives. As a result of this study, the order restricted likelihood ratio test was recommended over an ordered chi-square test for such situations provided the points at which jumps occur are known. If these points are unknown, the test based on Laplace's statistic was recommended. The performances of these tests were also considered for smooth alternatives.

15. "The Robustness and Precision of Parametric and Distribution-Free Tolerance Limits for Two Lifetime Distributions". A lower tolerance limit may also be interpreted as a lower confidence limit on a percentile, and it can also be related to a lower confidence limit on reliability. The robustness of an approximate tolerance limit for the gamma distribution was studied under a Weibull alternative, and the robustness of a Weibull tolerance limit was studied under a gamma alternative. Neither method was found to be very robust, but it was found that the Weibull limit is conservative for the gamma distribution with shape parameter greater than one. The amount of precision lost by using the robust distribution-free method based on an order statistic, compared to using the parametric gamma method was also studied.

16. "A Confidence Interval for Treatment Component-of-Variance with

Applications to Differences in Means of Two Exponential Distributions". There is no exact small-sample solution for setting confidence intervals for the treatment component in one factor components-of-variance problems, or for the problem of setting confidence intervals for the difference in means of two exponential distributions. A large number of approximate methods have been proposed for the components-of-variance problem. In a published study of nine of these methods, two have shown promise. The properties of these two as well as a third method, proposed by the authors, were investigated and shown to perform surprisingly well in the components-of -variance settings. The problem concerning differences of two exponential means is mathematically similar to the components-of-variance problem except that the parameter about which a confidence interval is to be built may take negative values. One may also wish to require a symmetry in the method so that the solution does not depend on the order in which the two samples are labelled. Adaptations of the above mentioned methods to the exponential means problem were given. It was demonstrated, by a Monte Carlo study, that two of the methods perform quite well for the exponential problem.

Status of Publications and Submissions 1984-88

1. "On the Asymptotic Behavior of the Mean Time Between Failures for Repairable Systems", The Proceedings of the International Conference on Reliability and Quality Control, (1986), pp.1-7.

2. "Tests for an Increasing Trend in the Intensity of a Poisson Process: A Power Study", Journal of the American Statistical Association, (1985), pp.419-422.

3. "Goodness-of-Fit Tests for the Weibull Distribution with Unknown Parameters and Heavy Censoring", Journal of Statistical Computation and Simulation, (1985), pp.213-225.

4. "Statistical Analysis of a Compound Exponential Failure Model", Journal of Statistical Computation and Simulation, (1986), pp.299-315.

5. "Tests of Equal Gamma Distribution Means with Unknown and Unequal Shape Parameters", Technometrics, (1988), pp.169-174.

6. "Approximate Tolerance Limits and Confidence Limits on Reliability for the Gamma Distribution", IEEE Transactions on Reliability, (1984), pp.184-187.

7. "Approximate Distributional Results Based on the Maximum Likelihood Estimators for the Weibull Distribution", Journal of Quality Technology, (1986), pp.174-181.

8. "Prediction Intervals for the Gamma Distribution", Proceedings of the 18th Symposium on the Interface of Computer Science & Statistics, Fort Collins, Colorado, March (1986).

9. "On the Mean Time Between Failures for Repairable Systems", IEEE Transactions on Reliability, (1986), pp.419-422.

10. "Statistical Analysis of a Compound Power-Law Model for Repairable Systems", IEEE Transactions on Reliability, (1987), pp.392-396.

11. "Reliability Test Plans for One-Shot Devices Based on Repeated Samples", in revision, Technometrics, (1987).

12. "Inferences on N in a Binomial Distribution", submitted to Communications in Statistics, Simulation and Computation, (1987).

13. "A Study of Parametric and Distribution-Free Tolerance Limits for the Gamma Distribution", submitted to the Journal of Quality Technology, (1987).

14. "Tests for Positive Jumps in the Intensity of a Poisson Process: A Power Study", submitted to IEEE Transactions on Reliability, (1988).

15. "The Robustness and Precision of Parametric and Distribution-Free Tolerance Limits for Two Lifetime Distributions", submitted to the IEEE Transactions on Reliability, (1988).

16. "A Confidence Interval for Treatment Component-of-Variance with Applications to Differences in Means of Two Exponential Distributions", to appear in the Journal of Statistical Computation and Simulation, (1988).